MECHANICAL PRINCIPLES

THIN WALLED VESSELS and THICK WALLED CYLINDERS

You should judge your progress by completing the self assessment exercises. These may be sent for marking at a cost (see home page).

When you have completed this tutorial you should be able to do the following.

- Define a thin walled cylinder.
- Solve circumferential and longitudinal stresses in thin walled cylinders.
- Solve circumferential and longitudinal stresses in thin walled spheres.
- Calculate the bursting pressure of thin walled cylinders and spheres.
- Define a thick walled cylinder.
- Solve circumferential, radial and longitudinal stresses in thick walled cylinders.
- Calculate changes in diameter and volume due to pressure.
- Solve problems involving the compression of fluids into pressure vessels.
- Solve problems involving interference fits between shafts and sleeves.

CONTENTS

1. THIN WALLED CYLINDER.

A cylinder is regarded as thin walled when the wall thickness t is less than 1/20 of the diameter D. When the wall is thicker than this, it is regarded as a thick wall and it is treated differently as described later.

Consider a cylinder of mean diameter D, wall thickness t and length L. When the pressure inside is larger than the pressure outside by p, the cylinder will tend to split along a length and along a circumference as shown in figures 1 and 2.

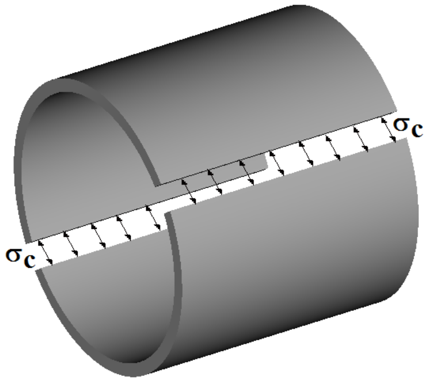

Figure 1

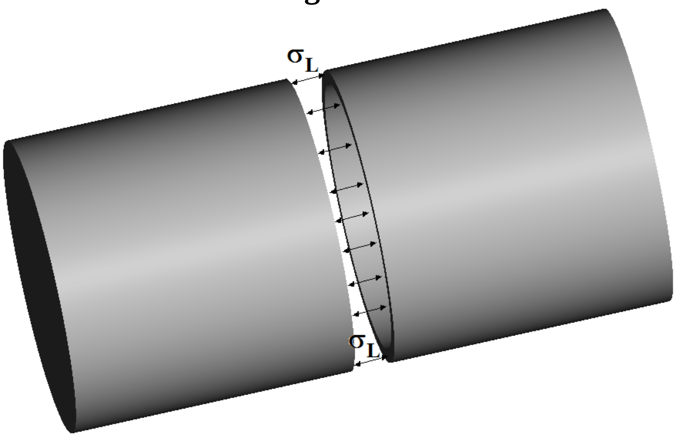

Figure 2

The stress produced in the longitudinal direction is σ_L and in the circumferential direction is σ_C. These are called the longitudinal and circumferential stresses respectively. The latter is also called the hoop stress.

Consider the forces trying to split the cylinder about a circumference (fig.2). So long as the wall thickness is small compared to the diameter then the force trying to split it due to the pressure is

$$F = pA = p\frac{\pi D^2}{4} \text{ ....(1.1)}$$

So long as the material holds then the force is balanced by the stress in the wall. The force due to the stress is

$$F = \sigma_L \text{ multiplied by the area of the metal} = \sigma_L \pi D t \text{ ....(1.2)}$$

Equating 1.1 and 1.2 we have

$$\sigma_L = \frac{pD}{4t} \text{....................................(1.3)}$$

Now consider the forces trying to split the cylinder along a length.
The force due to the pressure is

$$F = pA = pLD \text{ ....(1.4)}$$

So long as the material holds this is balanced by the stress in the material. The force due to the stress is

$$F = \sigma_C \text{ multiplied y the area of the metal} = \sigma_C 2Lt \text{ ....(1.5)}$$

Equating 1.4 and 1.5 we have

$$\sigma_C = \frac{pD}{2t} \text{ ....(1.6)}$$

It follows that for a given pressure the circumferential stress is twice the longitudinal stress.

WORKED EXAMPLE No.1

A cylinder is 300 mm mean diameter with a wall 2 mm thick. Calculate the maximum pressure difference allowed between the inside and outside if the stress in the wall must not exceed 150 MPa.

SOLUTION

The solution must be based on the circumferential stress since this is the largest.

σ_C = pD/2t = 150 MPa

p = 150 MPa x 2t/D = 150 x 2 x 0.002/0.3

p = 2 MPa

2. THIN WALLED SPHERE

A sphere will tend to split about a diameter as shown in fig.3

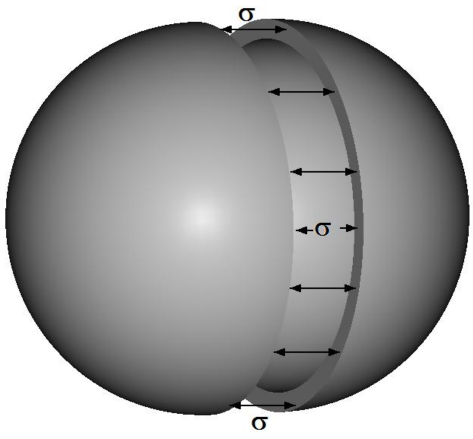

Figure 3

The stress produced in the material is equivalent to the longitudinal stress in the cylinder so

$$\sigma_C = \frac{pD}{4t} \quad \text{.................................(2.1)}$$

WORKED EXAMPLE No.2

Calculate the maximum allowable pressure difference between the inside and outside of a sphere 50 mm mean diameter with a wall 0.6 mm thick if the maximum allowable stress is 150 MPa.

SOLUTION

Using equation 2.1 we have

$\sigma = pD/4t = 150$ MPa

$p = 1.5\text{x}10^6 \text{ x } 4t/D = 1.5\text{x}10^6 \text{ x } 4 \text{ x } 0.0006/0.05 = 72$ kPa

SELF ASSESSMENT EXERCISE No.1

1. A thin walled cylinder is 80 mm mean diameter with a wall 1 mm thick. Calculate the longitudinal and circumferential stresses when the inside pressure is 500 kPa larger than on the outside. (Answers 10 MPa and 20 MPa).

2. Calculate the wall thickness required for a thin walled cylinder which must withstand a pressure difference of 1.5 MPa between the inside and outside. The mean diameter is 200 mm and the stress must not exceed 60 MPa. (Answer 2.5 mm)

3. Calculate the stress in a thin walled sphere 100 mm mean diameter with a wall 2 mm thick when the outside pressure is 2 MPa greater than the inside. (Answer -25 MPa).

3. VOLUME CHANGES

We will now look at how we calculate the changes in volume of thin walled vessels when they are pressurised.

CYLINDERS

Consider a small rectangular area which is part of the wall in a thin walled cylinder (figure 4).

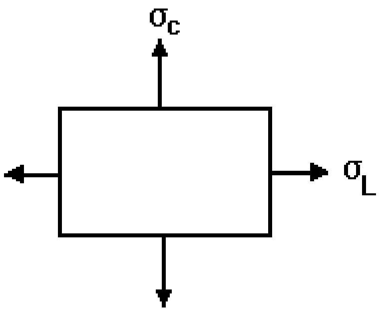

Figure 4

There are two direct stresses perpendicular to each other, σ_C and σ_L. From basic stress and strain theory (tutorial 1), the corresponding longitudinal strain is :

$$\varepsilon_L = \frac{1}{E}(\sigma_L - \nu\sigma_C)$$

E is the modulus of elasticity and ν is Poisson's ratio. Substituting σ_L= pD/4t and σ_C=pD/2t we have

$$\varepsilon_L = \frac{\Delta L}{L} = \frac{1}{E}\left(\frac{pD}{4t} - \nu\frac{pD}{2t}\right) = \frac{pD}{4tE}(1-2\nu) \text{.......................(3.1)}$$

The circumferential strain may be defined as follows.

ε_C= change in circumference/original circumference

$$\varepsilon_C = \frac{\pi(D+\Delta D) - \pi D}{\pi D} = \frac{\Delta D}{D}$$

The circumferential strain is the same as the strain based on diameter, in other words the diametric strain.

From basic stress and strain theory, the corresponding circumferential strain is :

$$\varepsilon_C = \frac{1}{E}(\sigma_C - \nu\sigma_L)$$

Substituting σ_L= pD/4t and σ_C=pD/2t we have

$$\varepsilon_C = \varepsilon_D = \frac{\Delta D}{D} = \frac{1}{E}\left(\frac{pD}{2t} - \nu\frac{pD}{4t}\right) = \frac{pD}{4tE}(2-\nu) \text{.......................(3.2)}$$

Now we may deduce the change in diameter, length and volume.

Original cross sectional area of cylinder = $A_1 = \pi D^2/4$

Original length = L_1 Original volume =$V_1 = A_1 L_1 = (\pi D^2/4)(L_1)$

New cross sectional area = $A_2 = \pi (D + \Delta D)^2/4$

New length = $L_2 = L + \Delta L$

New volume = $V_2 = A_2 L_2 = \{\pi(D + \Delta D)^2/4\}(L_1 + \Delta L)$

Change in volume = $\Delta V = V_2 - V_1$

Volumetric strain = $\varepsilon_V = \Delta V/V_1$

$$\varepsilon_V = \frac{\left(\frac{\pi(D+\Delta D)^2}{4}\right)(L_1+\Delta L) - \left(\frac{\pi D^2}{4}\right)L_1}{\left(\frac{\pi D^2}{4}\right)L_1} = \frac{\left(\frac{(D+\Delta D)^2}{4}\right)(L_1+\Delta L) - \left(\frac{D^2}{4}\right)L_1}{\left(\frac{D^2}{4}\right)L_1}$$

Dividing out and clearing brackets and ignoring the product of two small terms, this reduces to

$$\varepsilon_V = \frac{\Delta L}{L_1} + 2\frac{\Delta D}{D} = \varepsilon_L + 2\varepsilon_D \text{.............................(3.3)}$$

If we substitute equation 3.1 and 3.2 into this we find

$$\varepsilon_V = \frac{pD}{4tE}(5-4\nu) \text{.................................(3.4)}$$

WORKED EXAMPLE No.3

A cylinder is 150 mm mean diameter and 750 mm long with a wall 2 mm thick. It has an internal pressure 0.8 MPa greater than the outside pressure. Calculate the following.

i. The circumferential strain.
ii. The longitudinal strain.
iii. The change in cross sectional area.
iv. The change in length.
iv. The change in volume.

Take E = 200 GPa and ν = 0.25

SOLUTION

σ_C= pD/2t = 30 MPa σ_L= pD/4t = 15 MPa

ε_D= ΔD/D = (pD/4tE)(2 - ν) = 131.25 $\mu\varepsilon$

ΔD = 150 x 131.25 x 10^{-6} = 0.0196 mm D_2 = 150.0196 mm

$A_1 = \pi \times 150^2/4$ = 17671.1 mm^2 $A_2 = \pi \times 150.0196^2/4$ = 17676.1 mm^2

Change in area = 4.618 mm^2

ε_L= $\Delta L/L_1$ = (pD/4tE)(1 - 2ν) = 37.5$\mu\varepsilon$
ΔL= 750 x 37.5 x 10^{-6} = 0.0281 mm

Original volume = A_1L_1 = 13 253 600 mm^3
Final volume = A_2L_2 = 13 257 600 mm^3
Change in volume = 4000 mm^3
Check the last answer from equation 3.4
ε_V =(pD/4tE)(5 - 4ν) = 300 x 10^{-6}
Change in volume = $V_1 \times \varepsilon_V$ = 13 253 600 x 300 x 10^{-6} = 4000 mm^3

SPHERES

Consider a small rectangular section of the wall of a thin walled sphere. There are two stresses mutually perpendicular similar to fig. 4 but in this case the circumferential stress is the same as the longitudinal stress. The longitudinal strain is the same as the circumferential strain so equation 3.3 becomes

$\varepsilon_V = \varepsilon_D + 2\varepsilon_D$

$\varepsilon_V = 3\varepsilon_D$ ....(3.5)

The strain in any direction resulting from the two mutually perpendicular equal stresses is

$\varepsilon_D = (\sigma/E)(1-\nu)$

Hence $\varepsilon_V = 3(\sigma/E)(1-\nu)$ ....(3.6)

WORKED EXAMPLE No. 4

A sphere is 120 mm mean diameter with a wall 1 mm thick. The pressure outside is 1 MPa more than the pressure inside. Calculate the change in volume.

Take E = 205 GPa and ν= 0.26

SOLUTION

$\varepsilon_V = 3(\sigma/E)(1-\nu) = -324.87\mu\varepsilon$

(note the sphere shrinks hence the negative sign)

Original volume = $\pi D^3/6$ = 904778 mm^3

Change in volume = -904778 x 324.87 x 10^{-6} = -294 mm^3

WORKED EXAMPLE No. 5

In example No.3 the internal pressure is created by pumping water into the cylinder. Allowing for the compressibility of the water, deduce the volume of water at the outside pressure required to fill and pressurise the cylinder.

The bulk modulus K for water is 2.1 GPa.

SOLUTION

Initial volume of cylinder = V_1 = 13 253 600 mm^3 = volume of uncompressed water

Final volume of cylinder = V_2 = 13 257 600 mm^3 = volume of compressed water.

If V_2 was uncompressed it would have a larger volume V_3.

$V_3 = V_2 + \Delta V$ (all volumes refer to water).

From the relationship between pressure and volumetric strain we have

$\Delta V = pV_3/K = 0.8 \times 10^6 \times V_3 / 2.1 \times 10^9 = 380.9 \times 10^{-6} V_3$

$V_3 = 13\ 257\ 600 + 380.9 \times 10^{-6} V_3$

$0.9996 V_3 = 13\ 257\ 600$

$V_3 = 13\ 262\ 700\ mm^3$

This is the volume required to fill and pressurise the cylinder. The answer is not precise because the mean dimensions of the cylinder were used not the inside dimensions.

SELF ASSESSMENT EXERCISE No. 2

1. A cylinder is 200 mm mean diameter and 1 m long with a wall 2.5 mm thick. It has an inside pressure 2 MPa greater than the outside pressure. Calculate the change in diameter and change in volume.
   Take E = 180 GPa and ν = 0.3
   (Answers 0.075 mm and 26 529 mm^3)

2. A sphere is 50 mm mean diameter with a wall 0.5 mm thick. It has an inside pressure 0.5 MPa greater than the outside pressure. Calculate the change in diameter and change in volume.
   Take E = 212 GPa and ν= 0.25
   (Answers 0.0022 mm and 8.68mm^3)

3a. A thin walled cylinder of mean diameter D and length L has a wall thickness of t. It is subjected to an internal pressure of p. Show that the change in length ΔL and change in diameter ΔD are

$$\Delta L=(pDL/4tE)(1-2\nu) \quad \text{and} \quad \Delta D=(pD^2/4tE)(2-\nu)$$

b. A steel cylinder 2 m long and 0.5 m mean diameter has a wall 8 mm thick. It is filled and pressurised with water to a pressure of 3 MPa gauge. The outside is atmosphere. For steel E= 210 GPa and ν=0.3.
   For water K = 2.9 GPa.

   Calculate the following.
   i. The maximum stress. (93.75 MPa)
   ii. The increase in volume of the cylinder. (333092 mm^3)
   iii. The volume of water at atmospheric pressure required. (392 625 000mm^3)

4a. A thin walled sphere of mean diameter D has a wall thickness of t. It is subjected to an internal pressure of p. Show that the change in volume ΔV and change in diameter ΔD are

$$\Delta V=(3pDV/4tE)(1-\nu) \text{ where V is the initial volume.}$$

b. A steel sphere 2m mean diameter has a wall 20 mm thick. It is filled and pressurised with water so that the stress in steel 200 MPa. The outside is atmosphere. For steel E= 206 GPa and ν = 0.3. For water K = 2.1 GPa.

   Calculate the following.

   i. The gauge pressure (8 MPa)

   ii. The volume water required. (4.213×10^9 mm^3)

4. THICK CYLINDERS

The difference between a thin cylinder and a thick cylinder is that a thick cylinder has a stress in the radial direction as well as a circumferential stress and longitudinal stress. A rule of thumb is that radial stress becomes important when the wall thickness exceeds 1/20th of the diameter.

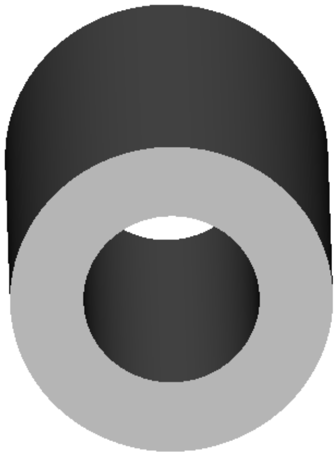

Figure 5

4.1 LAME'S THEORY

Consider a small section of the wall.

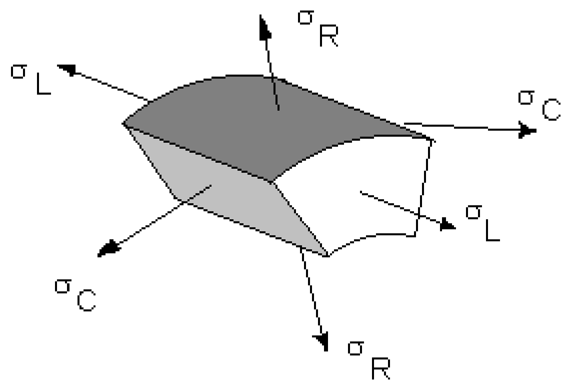

Figure 6

σ_L = Longitudinal stress

σ_R = Radial stress

σ_C = Circumferential stress

We have 3 stresses in mutually perpendicular directions, the corresponding strains are

$$\varepsilon_l = \frac{1}{E}\{\sigma_L - \nu(\sigma_R + \sigma_C)\}$$

$$\varepsilon_C = \frac{1}{E}\{\sigma_C - \nu(\sigma_L + \sigma_R)\}$$

$$\varepsilon_R = \frac{1}{E}\{\sigma_R - \nu(\sigma_C + \sigma_L)\}$$

Next consider the forces acting on a section of the wall.

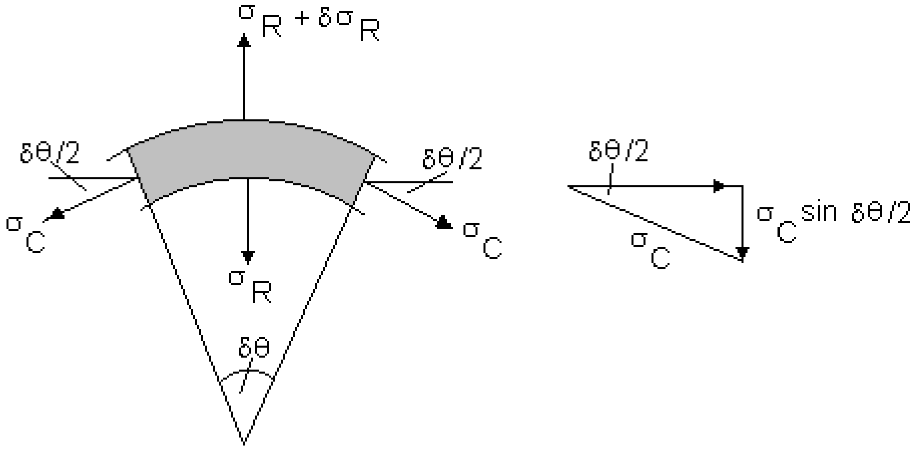

Figure 7

Balance the forces vertically (assuming 1 m of length).

Remember the length of an arc is radius x angle
The area of the top curved surface is $(r + \delta r)\delta\theta$ x 1
The area of the bottom curved surface is $r\delta\theta$ x 1

Remember Force is stress x area.
The vertical force up is $(\sigma_R + \delta\sigma_R)(r + \delta r)\delta\theta$
The vertical force down is $\sigma_R\, r\, \delta\theta + 2\,\sigma_C\, \delta r \sin\delta\theta/2$

Remember for small angles the sin is the same as the angle in radians.
$\sin\delta\theta/2 = \delta\theta/2$ Balancing the forces we have

$$(\sigma_R + \delta\sigma_R)(r + \delta r)\delta\theta = \sigma_R r\, \delta\theta + 2\sigma_C \delta r \frac{\delta\theta}{2}$$

This resolves down to $\dfrac{\delta\sigma_R}{\delta r} = \sigma_C - \sigma_R$

In the limit this becomes $\dfrac{d\sigma_R}{dr} = \sigma_C - \sigma_R$ ....(4.1)

Without proof, it can be shown that the longitudinal stress and strain are the same at all radii.
(The proof of this is a long piece of work and would detract from the present studies if given here).

The strain is given by

$$\varepsilon_L = \frac{1}{E}\{\sigma_L - \nu(\sigma_C + \sigma_C)\}$$

Since ε_L and σ_L are constant then it follows that $(\sigma_R + \sigma_C)$ = constant.
The solution is simplified by making the constant 2a

$$(\sigma_R + \sigma_C) = 2a$$
$$\sigma_C = 2a - \sigma_R \quad ....(4.2)$$

Substitute (4.2) into (4.1) and

$$\frac{rd\sigma_R}{dr} = 2a - \sigma_R - \sigma_R = 2a - 2\sigma_R$$

multiply all by r and rearrange

$$\frac{r^2 d\sigma_R}{dr} - 2ar = 2r\sigma_R$$

It can be shown that

$$\frac{d(r^2\sigma_R)}{dr} = \frac{r^2 d\sigma_R}{dr} + 2r\sigma_R$$

$$\frac{d(r^2\sigma_R)}{dr} = 2ar$$

$$(r^2\sigma_R) = \int 2ardr = ar^2 - b$$

where b is a constant of integration.

$$\sigma_R = a - \frac{b}{r^2}$$

$$\sigma_C = a + \frac{b}{r^2}$$

In order to solve problems, the constants a and b must be found from boundary conditions.

Remember: a boundary condition is a known answer such as knowing what the pressure or stress is at a given radius.

When atmospheric pressure acts on one side of the wall, it is best to use gauge pressure in the calculations. This makes atmospheric pressure zero and all other pressures are relative to it.

Remember: absolute pressure = gauge pressure + atmospheric pressure.

WORKED EXAMPLE No.6

A hydraulic cylinder is 100 mm internal diameter and 140 mm external diameter. It is pressurised internally to 100 MPa gauge. Determine the radial and circumferential stress at the inner and outer surfaces.
Take E = 205 GPa and ν = 0.25

SOLUTION

The boundary conditions are

Inner surface r = 50 mm σ_R = - 100 MPa (compressive)
Outer surface r = 70 mm σ_R = 0 MPa (compressive)

Substituting into Lame's equation we have
$\sigma_R = -100 \times 10^6 = a - b/r^2 = a - b/0.05^2$
$\sigma_R = 0 = a - b/r^2 = a - b/0.07^2$

Solving simultaneous equations b = 510 kN a = 104 MPa

Now solve the circumferential stress. $\sigma_c = a + b/r^2$

Putting r = 0.05 σ_c = 308 MPa
Putting r = 0.07 σ_c = 208 MPa

5. SOLID SHAFTS AND SLEEVES

In this section we will examine the stress and strain induced when a sleeve fits on a shaft with an interference fit.

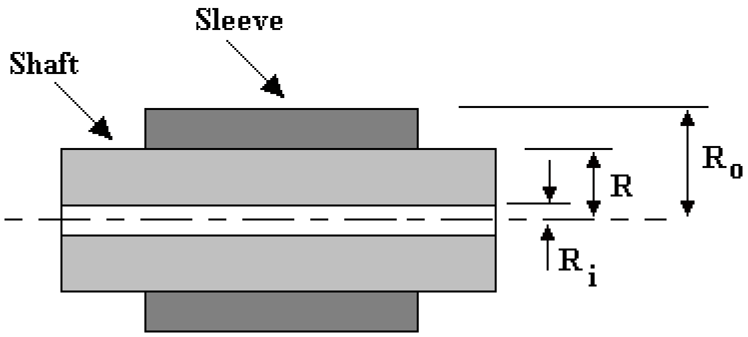

Figure 8

When the sleeve is fitted we assume here that a pressure p is exerted all over the surface of contact.

Fits consider the shaft. We will derive the equations as though the shaft was hollow with no pressure inside it and then put zero for the inside diameter.

$$\sigma_R = a - \frac{b}{r^2} \quad \text{At } r = R_i \quad \sigma_R = 0 \quad a = \frac{b}{R_i^2}$$

$$\text{At } r = R \quad \sigma_R = -p \quad \sigma_R = -p = \frac{b}{R_i^2} - \frac{b}{R_2}$$

$$-p = b\left(\frac{R^2 - R_i^2}{R_i^2 R^2}\right) \quad b = -p\left(\frac{R_i^2 R^2}{R^2 - R_i^2}\right)$$

$$a = -p\left(\frac{R^2}{R^2 - R_i^2}\right)$$

$$\sigma_c = a + \frac{b}{r^2}$$

$$\text{At } r = R \quad \sigma_c = -p\left(\frac{R^2}{R^2 - R_i^2}\right) - p\left(\frac{R_i^2}{R^2 - R_i^2}\right)$$

$$\sigma_c = -p\left[\frac{R^2 + R_i^2}{R^2 - R_i^2}\right]$$

Put $R_i = 0$ and $\boldsymbol{\sigma_c = -p}$

The strain in the circumferential direction = ε_c

$$\varepsilon_c = \frac{1}{E}(\sigma_c - \nu\sigma_R) = \frac{1}{E}(-p + \nu p) = \frac{p}{E}(\nu - 1)$$

$$\varepsilon_c = \frac{\Delta R}{R} \quad \Delta R = \frac{pR}{E}(\upsilon - 1)$$

This is the change in the outer diameter of the shaft. ν is Poison's ratio.

Next consider the sleeve.

$$\sigma_R = a - \frac{b}{r^2} \quad \text{At } r = R_o \quad \sigma_R = 0 \quad a = \frac{b}{R_o^2}$$

$$\text{At } r = R \ \sigma_R = -p \quad \sigma_R = -p = \frac{b}{R_o^2} - \frac{b}{R^2}$$

$$-p = b\left(\frac{R^2 - R_o^2}{R^2 R_o^2}\right) \quad b = -p\left(\frac{R^2 R_o^2}{R^2 - R_o^2}\right)$$

$$a = -p\left(\frac{R^2}{R^2 - R_o^2}\right)$$

$$\sigma_c = a + \frac{b}{r^2}$$

$$\text{At } r = R \quad \sigma_c = -p\left(\frac{R^2}{R^2 - R_o^2}\right) - p\left(\frac{R_o^2}{R^2 - R_o^2}\right)$$

$$\sigma_c = -p\left[\frac{R^2 + R_o^2}{R^2 - R_o^2}\right]$$

The strain in the circumferential direction = ε_c

$$\varepsilon_c = \frac{1}{E}(\sigma_c - \nu\sigma_R) = \frac{1}{E}\left(-p\left[\frac{R^2 + R_o^2}{R^2 - R_o^2}\right] + \nu p\right) = \frac{p}{E}\left(\nu - \left[\frac{R^2 + R_o^2}{R^2 - R_o^2}\right]\right)$$

$$\varepsilon_c = \frac{\Delta R}{R} \quad \Delta R = \frac{Rp}{E}\left(\nu - \left[\frac{R^2 + R_o^2}{R^2 - R_o^2}\right]\right)$$

This is the change in the inner diameter of the sleeve.

The decrease in radius of the shaft plus the increase in radius of the sleeve must add up to be the interference fit δ so adding the two ΔR values we get:

$$\delta = \frac{Rp}{E_1}\left(\nu_1 - \left[\frac{R^2 + R_o^2}{R^2 - R_o^2}\right]\right) + \frac{pR}{E_2}(\nu_2 - 1)$$

If the elastic constants are the same for both materials this simplifies to :

$$\delta = \frac{pR}{E}\left(2\nu - \left[\frac{R^2 + R_o^2}{R^2 - R_o^2}\right] - 1\right)$$

WORKED EXAMPLE No.7

A shaft has a diameter of 30.06 mm and is an interference fit with a sleeve 40 mm outer diameter, 30 mm inner diameter and 50 mm long. Calculate the force needed to slide the sleeve on the shaft if the coefficient of friction is 0.3. The elastic properties for both parts are the same with E = 205 GPa and Poisson's ratio = 0.25

Calculate the change in radius of the shaft and sleeve at the inside.

SOLUTION

$$\delta = \frac{pR}{E}\left(2\nu - \left[\frac{R^2 + R_o^2}{R^2 - R_o^2}\right] - 1\right) \quad R = 0.03 \text{ m} \quad R_o = 0.02 \quad \delta = 0.00003 \text{ m}$$

$$p = \frac{\delta E}{R}\frac{1}{\left(2\nu - \left[\frac{R^2 + R_o^2}{R^2 - R_o^2}\right] - 1\right)} = 133.5 \text{ MPa}$$

The normal force between the two surfaces of contact is N = pA

$A = 2\pi RL = 2\pi \times 0.015 \times 0.05 = 4.712 \times 10^{-3}\ m^2$

$N = 133.5 \times 10^6 \times 4.712 \times 10^{-3} = 629$ kN

Force to overcome friction $F = \mu R = 0.3 \times = 188.7$ kN

For the shaft

$$\Delta R = \frac{pR}{E}(\nu - 1) = \frac{133.5 \times 10^6 \times 0.015}{205 \times 10^9}(0.25 - 1) = -7.326 \times 10^{-6} \text{ m}$$

For the sleeve at the inside

$$\Delta R = \frac{Rp}{E_1}\left(\nu_1 - \left[\frac{R^2 + R_o^2}{R^2 - R_o^2}\right]\right) = 37.33 \times 10^{-6} \text{ m}$$

Check by adding 37.33 - 7.326 = 30μm the interference fit.

SELF ASSESSMENT EXERCISE No.3

1. A thick cylinder has an outer diameter of 150 mm and an inner diameter of 50 mm. The OUTSIDE is pressurised to 200 bar greater than the inside.

   Calculate the following.

   - The circumferential stress on the inside layer. (-45 MPa)
   - The circumferential stress on the outside layer. (-25 MPa)

2. A thick cylinder has an outside diameter of 100 mm and an inside diameter of 60 mm. It is pressurised until internally until the outer layer has a circumferential stress of 300 MPa.

   Calculate the pressure difference between the inside and outside. (266.6 MPa)

3. A thick cylinder is 100 mm outer diameter and 50 mm inner diameter. It is pressurised to 112 MPa gauge on the inside. Calculate the following.

   - The circumferential stress on the outside layer (74.64 MPa)
   - The circumferential stress on the inside layer (186.67 MPa)
   - The longitudinal stress (37.33 MPa)
   - The circumferential strain in the outside layer (314.9 $\mu\varepsilon$)
   - The circumferential strain in the inside layer (1.008×10^{-3})
   - The change in the inner diameter (0.05 mm)
   - The change in the outer diameter (0.031 mm)

   Take E = 205 GPa and ν = 0.27

4. A shaft has a diameter of 45.08 mm and is an interference fit with a sleeve 60 mm outer diameter, 45 mm inner diameter and 80 mm long. Calculate the force needed to slide the sleeve on the shaft if the coefficient of friction is 0.25. The elastic properties for both parts are the same with E = 200 GPa and Poisson's ratio = 0.3

   Calculate the change in radius of the shaft and sleeve at the inside.
   (p =111.2 MPa, F = 236.2 kN, ΔR_1=48.77 μm, ΔR_2 =-8.77 μm)